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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- The Effect of Launch Time on the Performance of a Solar Array/Battery Electrical Power System

TM- 67-1022-3

DATE- July 11, 1967

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AUTHOR(S)- W. W. Hough

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ABSTRACT

This report presents an analytical treatment of the subject of continuous power output from a solar array battery system on a space vehicle in near earth orbit as a function of several variables. Among the variables are launch time, mission time and duration, flight attitude, and array orientation with respect to the spacecraft. This analysis was performed as part of a trade-off study on power system configuration vs. flight attitude for the Apollo Applications Missions 1 and 2, and all numerical results are based on the fixed parameters of that set of missions.

Statistical analyses of the minimum angle between the sun line and the orbital plane, β, and of continuous power outputs have yielded average values of these quantities over the course of missions launched at specific times. Maximums, minimums, means, and variances of these averages have been found to evaluate the power system capability for a random launch time. This approach yields a basis for comparing power system configurations and flight attitudes that is more valid than a study of performance at the limits of β (in this case 0 and 51.95°) which occur infrequently, and perhaps never, over the course of a mission. For a spacecraft stabilized by gravity-gradient torque, optimum fixed array orientations have been found as a function of β.

For the AAP-1/AAP-2 28 day mission, the average β angle will vary between 9.54° and 35.28° depending on launch time. The yearly mean for a random launch time is 20.44°. Average continuous power output for electrical power system parameters assumed (6.07 KW sun orientated array output) will vary between 2.73 and 2.94 KW for an inertial attitude, with the mean over a year being 2.80 KW. If the spacecraft is gravity-gradient stabilized, the mean system output will be 1.60 KW or 57% of the mean available with an inertial attitude (neglecting spacecraft shadowing) if the array orientation is variable with mission time, and 1.49 KW or 53% if the array is fixed at an orientation that is optimum at the yearly mean of β. An array orientation exists that makes continuous power output almost independent of both launch and mission time, but drops the average continuous output to 1.36 KW or 48% of that available with an inertial orientation.

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SUBJECT: The Effect of Launch Time on the Performance of a Solar Array/Battery Electrical Power System - Case 600-3

DATE: July 11, 1967

FROM: W. W. Hough

TM-67-1022- 3

TECHNICAL MEMORANDUM

I. INTRODUCTION

This memorandum reports a study of the effect of launch time on the power output of a solar array/battery electrical power system mounted in various configurations to satellites in near-earth orbit. The study was performed as part of a trade-off on flight attitudes vs. power system configurations for Apollo Applications Mission 1 and 2 (AAP-1/AAP-2). The power for these combined missions is provided by CSM fuel cells plus a solar cell array/battery system on the Orbital Workshop.

The two attitudes under consideration for the AAP-1/AAP-2 configuration are gravity-gradient stabilized and inertially stabilized. The latter gives (as will be shown later) a higher solar array output, but is not without penalties. The quasi-inertial method,[1] in which the spacecraft tumbles passively about an axis normal to the orbital plane at an average rate equal to the negative of the orbital rate, requires reaction control system propellant for frequent initialization of attitude and attitude rates. Active inertial stabilization by control moment gyros has the penalty of the weight of those gyros. Both methods of inertial stabilization require more power to run the control systems than the gravity-gradient stabilization mode, which is completely passive once established.

The Introduction to this point has indicated some of the factors that must be considered in the total trade-off study. From this point, discussion will be limited to the specific subject of the power available from solar arrays as a function of configuration relative to the satellite, flight attitude, and launch time. The equations developed are applicable for any circular earth orbit, but numerical results are based on the fixed parameters of the AAP-1/AAP-2 mission (i.e., 28 1/2° inclined orbit of 270 nautical mile altitude).

In a gravity-gradient stabilized attitude, the power output of a fixed solar array is heavily dependent on the angle between

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[1]For a complete discussion of the quasi-inertial method of spacecraft stabilization, see Bellcomm TR-67-600-3-1, B. D. Elrod, "Quasi-Inertial Stabilization of the AAP 1/2 Cluster Configuration", April 14, 1967.

the solar vector and the orbital plane. This angle also influences the power available from an inertially stabilized, or equivalently a sun-oriented array, because of variation in earth shadow time. This angle is time-variable within limits determined by the season of the year and orbital inclination, and its initial value at launch is a function of variables such as the time of day and time of year of launch and the launch azimuth. The statistical study of the mission average functions of the sun-line-orbital plane angle presented in this memorandum can be used for thermal balance analyses as well as analyses of power output of solar arrays.

II. THE EQUATION FOR THE ANGLE BETWEEN THE SUN LINE AND THE ORBITAL PLANE

The position of any circular earth orbit with respect to the sun can be determined by the specification of four angles shown in Figure 1. The required definitions of axes and angles are:

- ϒ: Along the line of nodes of the ecliptic and equatorial planes with origin at the center of the earth, 0, and positive toward the sun at the vernal equinox. ϒ is space fixed.

- S: The solar vector directed from the center of the earth toward the sun.

- γ: The angle between ϒ and S measured in the ecliptic. γ is zero at vernal equinox and increases with time.

- e: The angle between the ecliptic and equatorial planes. A negative rotation about ϒ through e will transform a set of ecliptic coordinates to equatorial coordinates. The angle e is taken at a fixed value of 23°27'.

- χ: Along the line of nodes of the equatorial and orbital planes with origin at the center of the earth and positive toward the ascending node.

- Ω: The angle between ϒ and χ measured in the equatorial plane. Ω is positive as shown in Figure 1, and increases as χ becomes more easterly of ϒ. (Ω can also be thought of as the right ascension of the ascending node of the orbit.)

- i: The angle between the equatorial and orbital planes, or orbital inclination.

- Z: The normal to the orbital plane, positive toward the north.

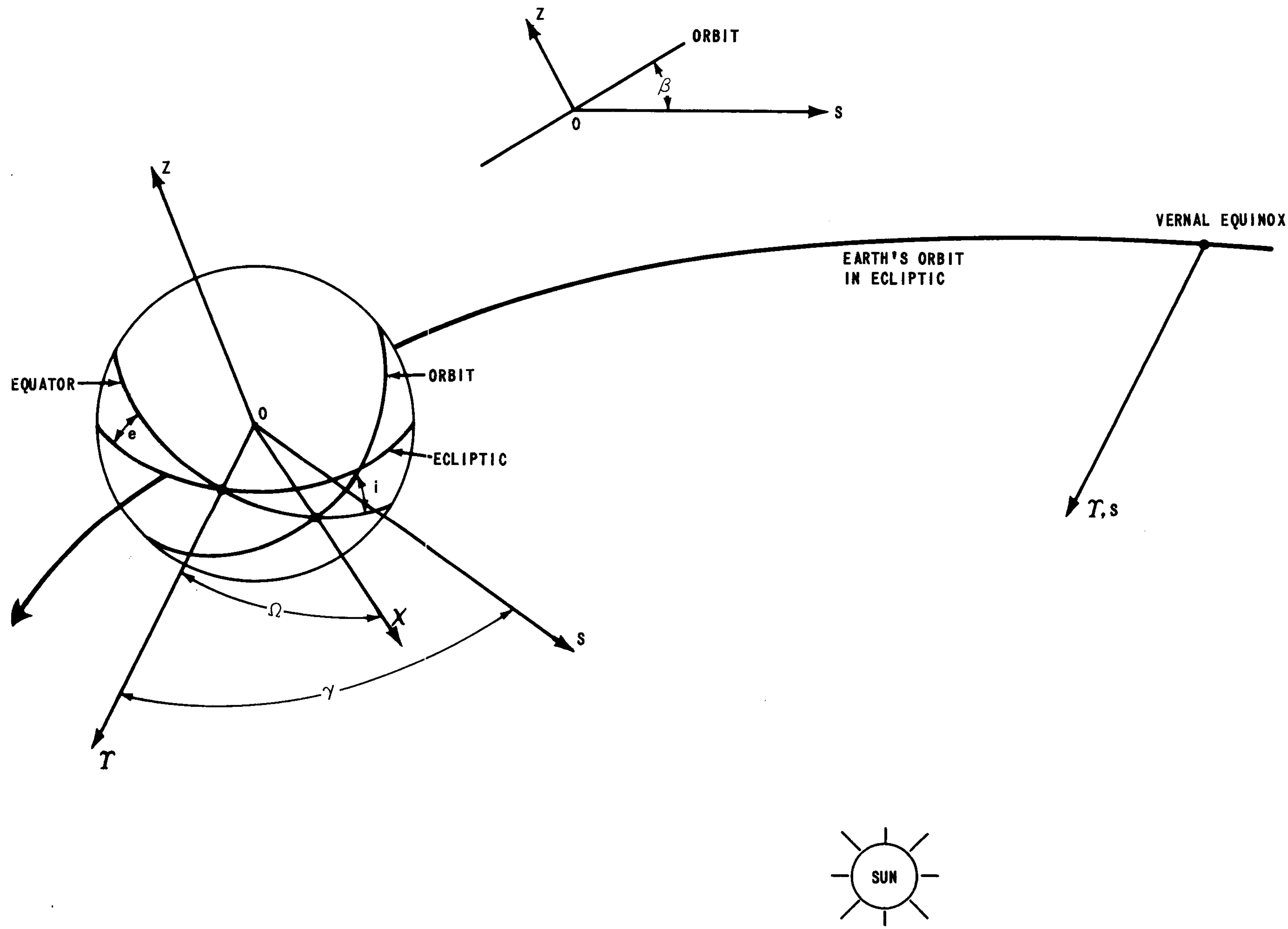

FIGURE I: GEOMETRY FOR DETERMINATION OF THE ANGLE BETWEEN THE SUN-LINE AND THE ORBITAL PLANE

β: The angle between the orbital plane and the solar vector, S. If we view the orbit as projected in the plane determined by S and Z, it will appear as a line (see sketch at the top of Figure 1.) β is the angle shown and is positive when the angle S-0-Z is obtuse and negative when that angle is acute (i.e., β = angle S-0-Z - 90°).

The functional relationship between β and the four other angles just defined is:

$$\sin\beta = \sin\gamma(\sin i \cos\Omega \cos e - \cos i \sin e) - \sin i \sin\Omega \cos\gamma \tag{1}$$

The derivation of this equation, with one difference, will be given in a Bellcomm memorandum[2] and will not be repeated here. (The difference is that in that derivation the autumnal equinox was taken as reference, and this results in a positive rather than a negative "$\sin\gamma \cos i \sin e$" term in the above equation because the rotation through e is a positive rotation.)

The angle e is always fixed, and the angle i is fixed for any given mission. Therefore the equation for $\sin\beta$ can be written:

$$\sin\beta = A \sin\gamma \cos\Omega + B \sin\Omega \cos\gamma + C \sin\gamma \tag{2}$$

where $A = \sin i \cos e$

$B = -\sin i$

$C = -\cos i \sin e$

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[2] Elrod, B. D., "Derivation of Expression for Angle Between Sun Line and Orbit Plane", Bellcomm Memorandum for File, to be published.

The angles γ and Ω are both time dependent. Three times must be specified in the investigation of the behavior of β and its trigonometric functions.

t_o: the time of launch measured from the time of the vernal equinox

t : mission time measured from t_o ($t = 0$ at t_o)

τ_L: the local time of launch at the launch site.

If $\gamma = \gamma_o$ at t_o, then:

$$\gamma = \gamma_o + \dot{\gamma}t \tag{3}$$

where $\dot{\gamma} = 360^\circ$/solar year = 0.985647365°/day,

and if $\Omega = \Omega_o$ at t_o, then:

$$\Omega = \Omega_o + \dot{\Omega}t \tag{4}$$

where $\dot{\Omega}$ is the time rate of change of Ω. The major component of $\dot{\Omega}$ is due to the earth's oblateness. An approximate equation for $\dot{\Omega}$ is:[3]

$$\dot{\Omega} = - \frac{J_2 R^2 \mu^{1/2} \cos i}{(R + H)^{7/2}} \tag{5}$$

[3]Blitzer, Leon, "On the Motion of a Satellite in the Gravitational Field of the Oblate Earth", Space Technology Laboratories - GM-TM-0165-00279, September 5, 1958.

where $J_2 = 1.6234 \times 10^{-3}$

R = radius of the earth, or 3443.9 nautical miles (NM)

H = altitude of the orbit (NM)

μ = earth's gravitational constant = 4.68427×10^{14} NM^3/day^2

i = orbit inclination

With all known constants entered, the above equation becomes:

$$\dot{\Omega} = -\frac{2.3876 \times 10^{13} \cos i}{(3443.9 + H)^{1/2}} \text{ degrees/day} \tag{6}$$

The value of γ at launch is:

$$\gamma_o = \dot{\gamma} t_o \tag{7}$$

The specification of Ω_o is a much more complex problem. Referring to Figure 1, we can observe that the local time of day (12.50 = 12:30 hours) at Υ is

$$\tau_{\Upsilon} = 36.00 - \left[\frac{\gamma}{360^\circ}\right]\left[24 \text{ hrs.}\right] - n(24.00) \tag{8}$$

where n is 1 if $0^\circ \leq \gamma < 180^\circ$ and 0 if $180^\circ \leq \gamma \leq 360^\circ$ and simply makes $0 < \tau_{\Upsilon} \leq 24.00$. If Ω is specified, we can find the local time of day at the ascending node of the orbital plane (χ):

$$\tau_\chi = \tau_T + \left[\frac{\Omega}{360^\circ}\right]\left[24 \text{ hrs.}\right] - m(24.00)$$

or

$$\tau_\chi = 36.00 - \left[\frac{\gamma}{360^\circ}\right]\left[24 \text{ hrs.}\right] + \left[\frac{\Omega}{360^\circ}\right]\left[24 \text{ hrs.}\right] - q(24.00) \quad (9)$$

where m and q are integral multipliers like n that serve to satisfy the requirement that $0 < \tau_\chi \leq 24.00$. Note that $\Omega = \gamma$ if $\tau_\chi = 12.00$ (i.e., noon).

At t_o, that is at launch, the local time at the launch site is τ_L. For a launch from the northern hemisphere, the longitude of the launch site will be between 0 and 180° east of the ascending node, χ. Figure 2 shows the geometry necessary to determine the angle between χ and the longitude of the launch site.

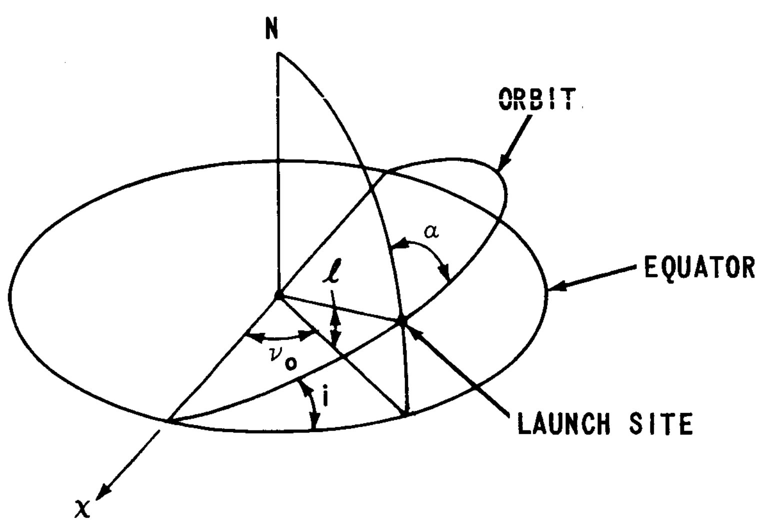

FIGURE 2

Definitions of the angles required are:

ν_o = angle between χ and the longitude of the launch site, at launch

ℓ = latitude of launch site

α = launch azimuth

By spherical trigonometry:

$$\cos \nu_o = \frac{\cos \alpha}{\sin i} \tag{10}$$

Launch azimuth is related to inclination by:

$$\sin \alpha = \frac{\cos i}{\cos \ell} \tag{11}$$

It is important to note that α in (11) is double valued, (i.e., two launch azimuths, varying equal amounts north and south from due east, can be used to establish a given orbital inclination from a certain launch latitude.) In using (11) it is therefore necessary to know whether the launch azimuth is greater or less than 90^o. The local time of day at the launch site is

$$\tau_L = \tau_{\chi_o} + \left[\frac{\nu_o}{360^o}\right] \left[24 \text{ hrs.}\right] - r(24.00)$$

or

$$\tau_L = 36.00 - \left[\frac{\gamma_o}{360^\circ}\right]\left[24 \text{ hrs.}\right] + \left[\frac{\Omega_o}{360^\circ}\right]\left[24 \text{ hrs.}\right] + \left[\frac{\nu_o}{360^\circ}\right]\left[24 \text{ hrs.}\right] - s(24.00) \qquad (12)$$

where r and s are integral multipliers that serve to satisfy the requirement that $0 < \tau_L \leq 24.00$. Zero subscripts have been added to γ and Ω to designate the values of these angles at t_o. With τ_L specified, ν_o calculated by (10), and γ_o evaluated by (7), equation (12) can be solved for Ω_o

$$\Omega_o = (\tau_L - 12.00)\ 15^\circ/\text{hr.} + \gamma_o - \nu_o \qquad (13)$$

Substitution of (3) and (4) in (2) gives

$$\sin\beta = A \sin(\gamma_o + \dot{\gamma}t)\cos(\Omega_o + \dot{\Omega}t) + B\sin(\Omega_o + \dot{\Omega}t)\cos(\gamma_o + \dot{\gamma}t) + C\sin(\gamma_o + \dot{\gamma}t) \qquad (14)$$

This equation can be put into a more usable form for study of $\sin\beta$ as a function of t:

$$\begin{aligned}\sin\beta = \; & J \sin(\gamma_o + \Omega_o) \times \cos\dot{\lambda}t \\ & + J \cos(\gamma_o + \Omega_o) \times \sin\dot{\lambda}t \\ & + K \sin(\gamma_o - \Omega_o) \times \cos\dot{\zeta}t \\ & + K \cos(\gamma_o - \Omega_o) \times \sin\dot{\zeta}t \\ & + C \sin \gamma_o \times \cos\dot{\gamma}t \\ & + C \cos \gamma_o \times \sin\dot{\gamma}t \end{aligned} \tag{15}$$

where: $J = \frac{A+B}{2}$, $K = \frac{A-B}{2}$, $\dot{\lambda} = \dot{\gamma}+\dot{\Omega}$, and $\dot{\zeta} = \dot{\gamma}-\dot{\Omega}$.

The left sides of all terms in (15) are constants for any specific launch time, with γ_o evaluated by (7) and Ω_o by (13). The right sides of all terms are harmonic functions of time.

III. PROPERTIES OF THE SUN LINE - ORBITAL PLANE ANGLE FOR AAP-1/AAP-2

The AAP-1/AAP-2 spacecraft will be inserted into a 28 1/2^o inclined circular orbit at a 270 nautical mile altitude using a 90^o launch azimuth from Cape Kennedy. The planned mission duration is 28 days. Although a scheduled launch date exists, launch time shall be treated as perfectly random. The limits on β for the 28 1/2^o orbit are $\pm$ 51^o 57'. However, negative values of β are no different from the spacecraft's point of view than positive values. A fixed solar array will receive the same incident sunlight at $\beta = -20^o$ as it will at $\beta = +20^o$. For some optimized solar array orientations for the gravity gradient stabilized attitude, a 180^o roll of the spacecraft is required to meet this condition, but there is no constraint against performing such a maneuver. The outputs of all arrays considered are dependent on the absolute value of β and its trigonometric functions.

Figure 3 shows the actual variation of $|\beta|$ with mission time for two cases. The case of launch on December 9 was picked so that β would reach its maximum limit midway through the 28 day AAP-1/AAP-2 mission. With this launch time and mission duration, β will never be zero. The second case was arbitrarily chosen from among the set used in the computer study. The February 10 launch gives a β equal to zero twice during the mission.

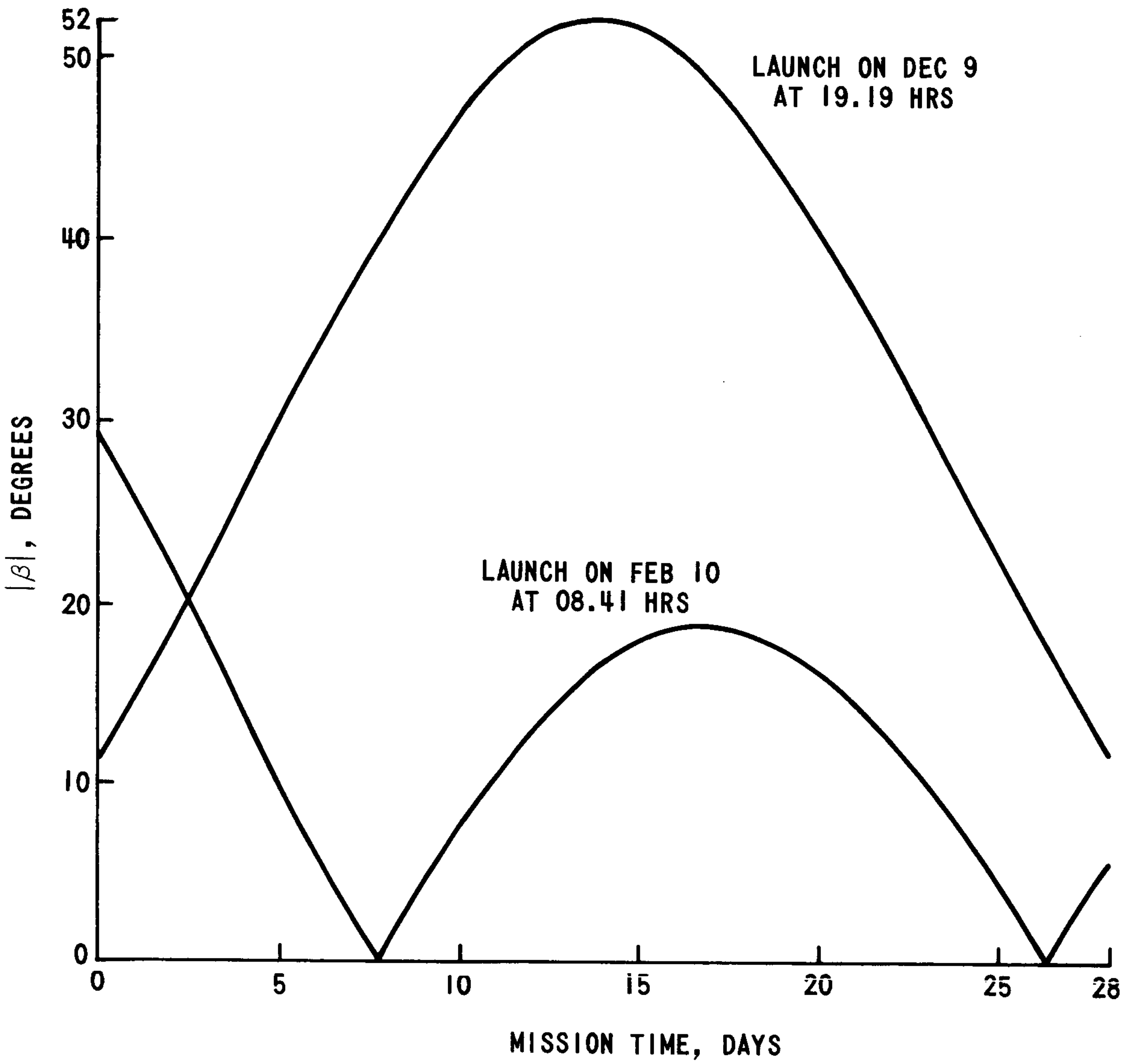

FIGURE 3

In comparing solar array configurations, the outputs of which are dependent on $|\beta|$, the average of the power over the mission will be used as a criteria. As a precursor to the study of average power outputs, it is useful to examine the average values of $|\beta|$; $\sin|\beta| = |\sin\beta|$; and $\cos|\beta| = |\cos\beta| = \cos\beta$, over the mission as a function of launch time. The expressions used are:

$$|\beta|_{avg}(t_o,\tau_L) = \left(\int_0^T |\beta(t_o,\tau_L,t)|dt\right) \Big/ T \qquad (16)$$

$$|\sin\beta|_{avg}(t_o,\tau_L) = \left(\int_0^T |\sin\beta(t_o,\tau_L,t)|dt\right) \Big/ T \qquad (17)$$

$$(\cos\beta)_{avg}(t_o,\tau_L) = \left(\int_0^T \cos\beta(t_o,\tau_L,t)dt\right) \Big/ T \qquad (18)$$

where T is total mission duration, i.e., 28 days. As there are no convenient closed form expressions for any of the above integrals, the calculations have been performed numerically. Approximately 1000 launch times (which set t_o and τ_L) have been used. The results are summarized in the following table, which lists the maximum and minimum of $|\beta|_{avg}$ defined by (16), and the associated values of $|\sin\beta|_{avg}$* and $(\cos\beta)_{avg}$ and the mean and variance** of these quantities for a random launch time.

	$\|\beta\|_{avg}$	$\|\sin\beta\|_{avg}$	$(\cos\beta)_{avg}$
MAXIMUM	35.28°	.564	.796
MINIMUM	9.54°	.164	.979
MEAN	20.44°	.337	.907
VARIANCE	6.97°	.108	.052

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* Note that $\sin(|\beta_{avg}|) \neq |\sin\beta|_{avg}$ and similarly with the cosine.

**Variance is defined as $\sqrt{\dfrac{\sum_{i=1}^{n} (\beta_i - \beta_{avg})^2}{n-1}}$

The maximum $|\beta|_{avg}$ will occur if the vehicle is launched 14 days prior to winter solstice, on December 9, at 19.19 hours KSC time. There is a similar time on June 9, which is 14 days prior to summer solstice, which also gives the initial condition to make $|\beta|_{avg}$ maximum. The minimum $|\beta|_{avg}$ is much more difficult to pick. The one given occurs with launch on January 21 at 6:00 hours. Several of the launch times (for example, about 6:00 a.m. for January dates and 6:00 p.m. for July dates) used in the computer study give a $|\beta|_{avg}$ between 9.5^o and 10^o.

Figure 4 is a plot of $|\beta|_{avg}$ for the 28 days following launch at progressive times from October 24 through October 26. The large variation and approximate daily periodicity indicate the heavy dependence of the $|\beta|$ history on the time of day of launch, (or in other words, the rate of the earth's rotation about its polar axis). In Figure 4, the daily mean of $|\beta|_{avg}$ for launch on these three days is also shown. Figure 5 is a plot of this same daily mean of $|\beta|_{avg}$ for the 28 day mission vs. launch date for the entire year. The mean of that curve, which is purely periodic, has the fixed value of 20.44^o as given in the table.

IV. SOLAR ARRAY POWER OUTPUT

For the purpose of this analysis it is assumed that when a spacecraft is in sunlight, the power output from a planar solar array is equal to the power output when the array is facing directly toward the sun times cosine of the angle between the sun-line and the outward normal to the array* when the cosine is positive. The output is zero when the cosine is negative. If the spacecraft is in the earth's shadow, the output is zero.

A. Gravity-Gradient Stabilized Attitude

Figures 6 and 7 show the geometry necessary to determine the cosine of the angle between the sun-line and the outward normal to an array fixed to a spacecraft in a gravity-gradient stabilized attitude. In this attitude, the spacecraft axis of minimum moment of inertia is in the orbital plane and aligned with the local vertical.

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*Secondary effects, such as variation of power as a function of panel temperature is beyond the scope of this memorandum.

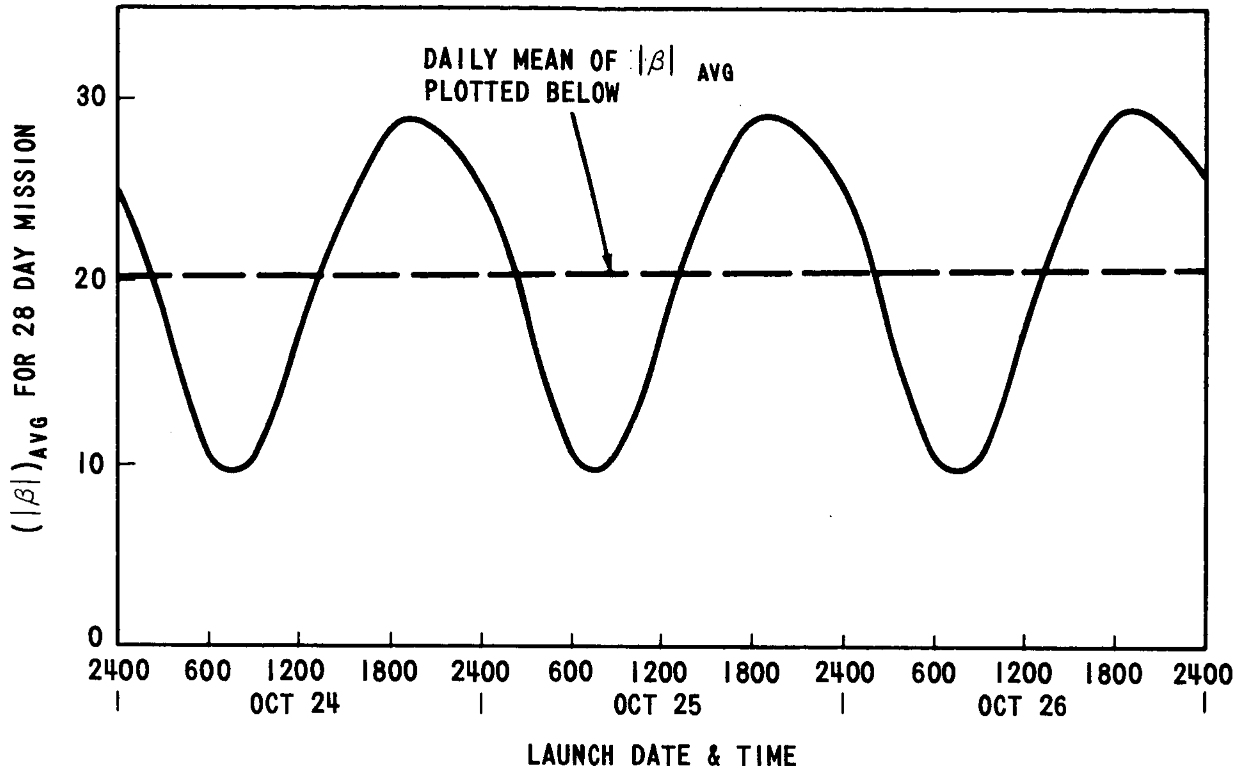

FIGURE 4

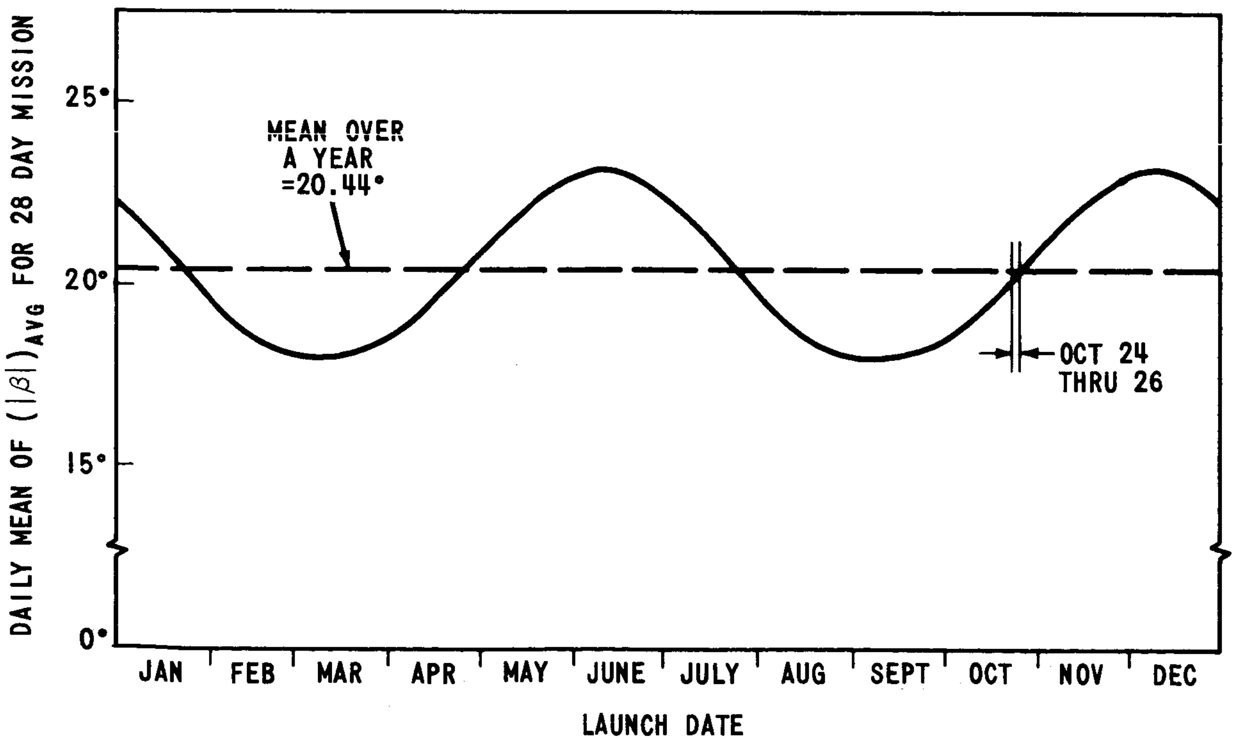

FIGURE 5

Specification of two angles is required to describe the orientation of the array with respect to the orbital plane and local vertical. These angles are shown in Figure 6.

ψ: The fixed angle between the line formed by the intersection of the plane of the array and the orbital plane and the tangent to the orbit. ψ is therefore the angle between the local vertical (positive away from the earth) and the projection of the outward normal to the array, $\overline{N}$, in the orbital plane. ψ is positive as shown.

θ: The fixed angle between the plane of the array and the orbital plane. The true view of θ is shown in the S-0-Z plane in Figure 6. The spacecraft is at the position where $\overline{N}$ is parallel to the S-0-Z plane and therefore the view of the array is a line.

Additional definitions required for Figures 6 and 7 are:

η: The angle that specifies the spacecraft position in the orbit. η is zero at noon, noon occurring when the spacecraft is in the S-0-Z plane on the sun side of the orbit, as shown in Figure 6. η is positive in the direction of the orbital velocity vector.

ϕ: The angle between the outward normal to the solar array, $\overline{N}$, and the solar vector, S.

As the spacecraft progresses around the orbit, the vector $\overline{N}$ (with origin at 0, the center of the earth) travels on the surface of a cone. This cone is centered on the normal to the orbital plane, Z, and has a half angle equal to θ. The cosine of ϕ will be zero when $\overline{N}$ is along the intersection of this cone and a plane that is normal to the sun line. This

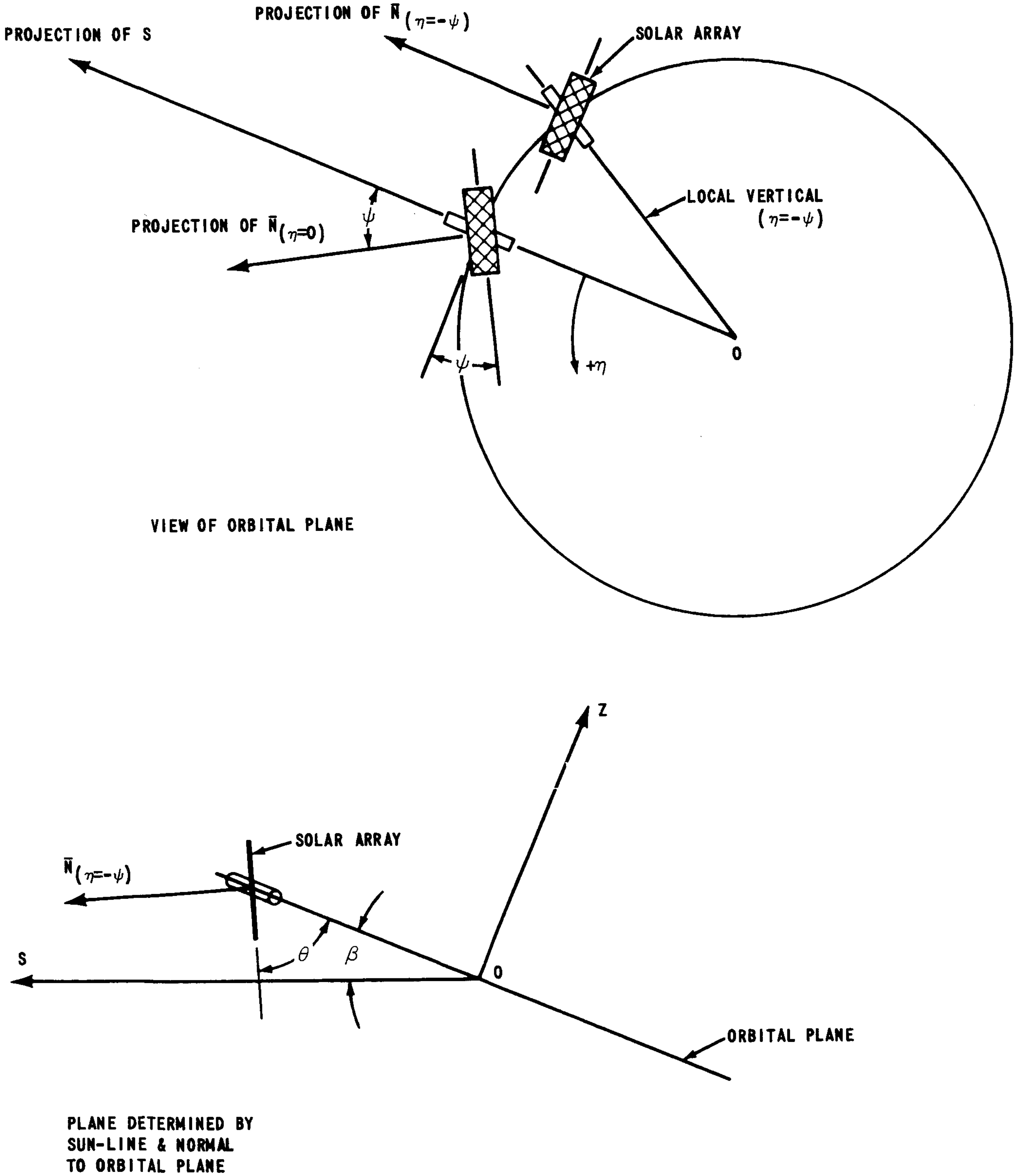
PROJECTION OF $\bar{N}_{(\eta=-\psi)}$
PROJECTION OF S
SOLAR ARRAY
LOCAL VERTICAL $(\eta=-\psi)$
PROJECTION OF $\bar{N}_{(\eta=0)}$
ψ
ψ
$+\eta$
0
VIEW OF ORBITAL PLANE
Z
SOLAR ARRAY
$\bar{N}_{(\eta=-\psi)}$
S
θ
β
0
ORBITAL PLANE
PLANE DETERMINED BY
SUN-LINE & NORMAL
TO ORBITAL PLANE

FIGURE 6

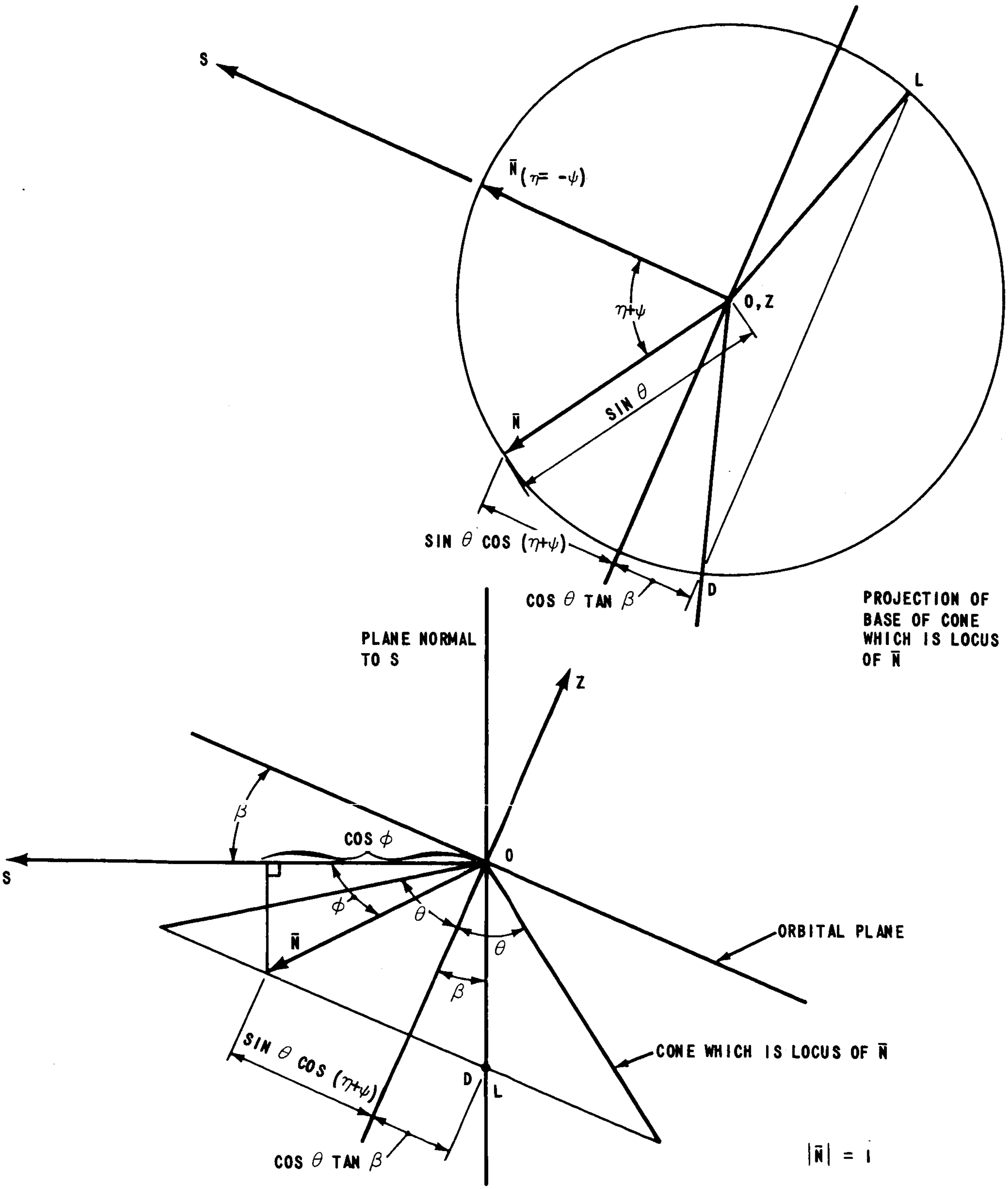
S
L
$\bar{N}(\eta = -\psi)$
$\eta+\psi$
O, Z
SIN θ
$\bar{N}$
SIN θ COS $(\eta+\psi)$
COS θ TAN β
D
PROJECTION OF BASE OF CONE WHICH IS LOCUS OF $\bar{N}$
PLANE NORMAL TO S
Z
β
COS ϕ
O
S
ϕ
θ
$\bar{N}$
θ
β
ORBITAL PLANE
CONE WHICH IS LOCUS OF $\bar{N}$
SIN θ COS $(\eta+\psi)$
D
L
COS θ TAN β
$|\bar{N}| = 1$

FIGURE 7

intersection is shown as the triangle D-0-L in Figure 7. When $\overline{N}$ is in the half-space on the sun-side of this plane, solar array output is positive, (neglecting for the moment earth shadowing). When $\overline{N}$ is in the half-space on the side of this plane away from the sun, array output is zero. If θ is less than β,* the cone will never intersect the plane normal to the sun line and $\overline{N}$ will always lie in the sun-side half-space. The projection of $\overline{N}$ in the base of the cone (or in the orbital plane) will make an angle of $(\eta + \psi)$ to the projection of the solar vector in that same plane.

The expression for cos ϕ in terms of β, θ, η, and ψ can be derived directly from Figure 7. If $\overline{N}$ is a unit vector, the component of $\overline{N}$ in the direction of S is cos ϕ. The length of the projection of $\overline{N}$ in the plane of the base of the cone, its locus, is sin θ. When viewed in the S-0-Z plane, its apparent length is $\sin\theta\cos(\eta + \psi)$. The height of the cone is cos θ. The distance between the normal to the orbital plane, Z, and the line D-L measured in the plane of the base of the cone and parallel to the S-0-Z plane is $\cos\theta\tan\beta$ as shown. The total distance between the tip of the unit vector $\overline{N}$ and the line DL, as measured in the base of the cone and parallel to the S-0-Z plane is therefore $\sin\theta\cos(\eta + \psi) + \cos\theta\tan\beta$. As the base of the cone makes the angle β with the solar vector, S, the projection along S of this distance is the distance times cos β. Therefore:

$$\cos\phi = \sin\theta\cos\beta\cos(\eta + \psi) + \cos\theta\sin\beta \tag{19}$$

Equation (19) can be solved for the values of η at which $\overline{N}$ passes into and out of the dark-side half-space, i.e., η_D and η_L respectively. This occurs when $\cos\phi = 0$; therefore:

$$\cos\left(\eta_{\substack{L\\D}} + \psi\right) = -\frac{\tan\beta}{\tan\theta} \tag{20}$$

---

* β can be considered as a fixed angle over the time for one orbit.

Defining $\xi = \cos^{-1}\left(-\frac{\tan\beta}{\tan\theta}\right)$, we note that ξ exists only if $\beta \leq \theta$.* This is consistent with previous discussion of Figure 6. If ξ exists, it is double valued, and:

$$\eta_D = \xi - \psi \qquad \text{for } \frac{\pi}{2} \leq \xi \leq \pi \tag{21}$$

$$\eta_L = \xi - \psi \qquad \text{for } -\pi \leq \xi \leq -\frac{\pi}{2} \tag{22}$$

B. Inertial Attitude**

In sunlight the power output of a solar array on an inertially oriented vehicle is directly proportional to the cosine of the fixed angle ϕ (defined as in the previous section). If the array is sun oriented, $\cos\phi = 1$, which is optimum.

C. Shadowing by the Earth

In the case of the gravity-gradient stabilized attitude, power output can go to zero at the limits η_D and η_L specified by equations (21) and (22), or either or both of these might be replaced by the angle's η_{DUSK} and η_{DAWN} at which the satellite enters and exits from the earth's shadow. In the case of the inertially oriented spacecraft, η_{DAWN} and η_{DUSK} alone determine the limits on power output. For a circular orbit, η_{DAWN} and η_{DUSK} are functions of only the orbital altitude and the sun line-orbital plane angle, β. This relationship can be written with the help of Figure 8. Three views are shown: a view of the orbital plane looking down at the top of the figure, a view of the S-O-Z plane at the bottom left, and a view looking directly down the sun line, S, at the bottom right. Point T is on the surface of the earth and is the point at which the last ray of sunlight is tangent as the satellite moves into the shadow.

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* In this discussion, β is limited to values between 0 and 90° for the same reasons that we took the absolute value of β in our previous considerations. Under this condition, we can observe from Figure 7 that the same limits on θ will provide at least a half orbit of sunlight, whereas $\theta > 90°$ will not.

**In subsequent discussion, all results in the "inertial attitude" category are equally applicable to a two-axis articulating, sun tracking array on a non-inertially oriented vehicle.

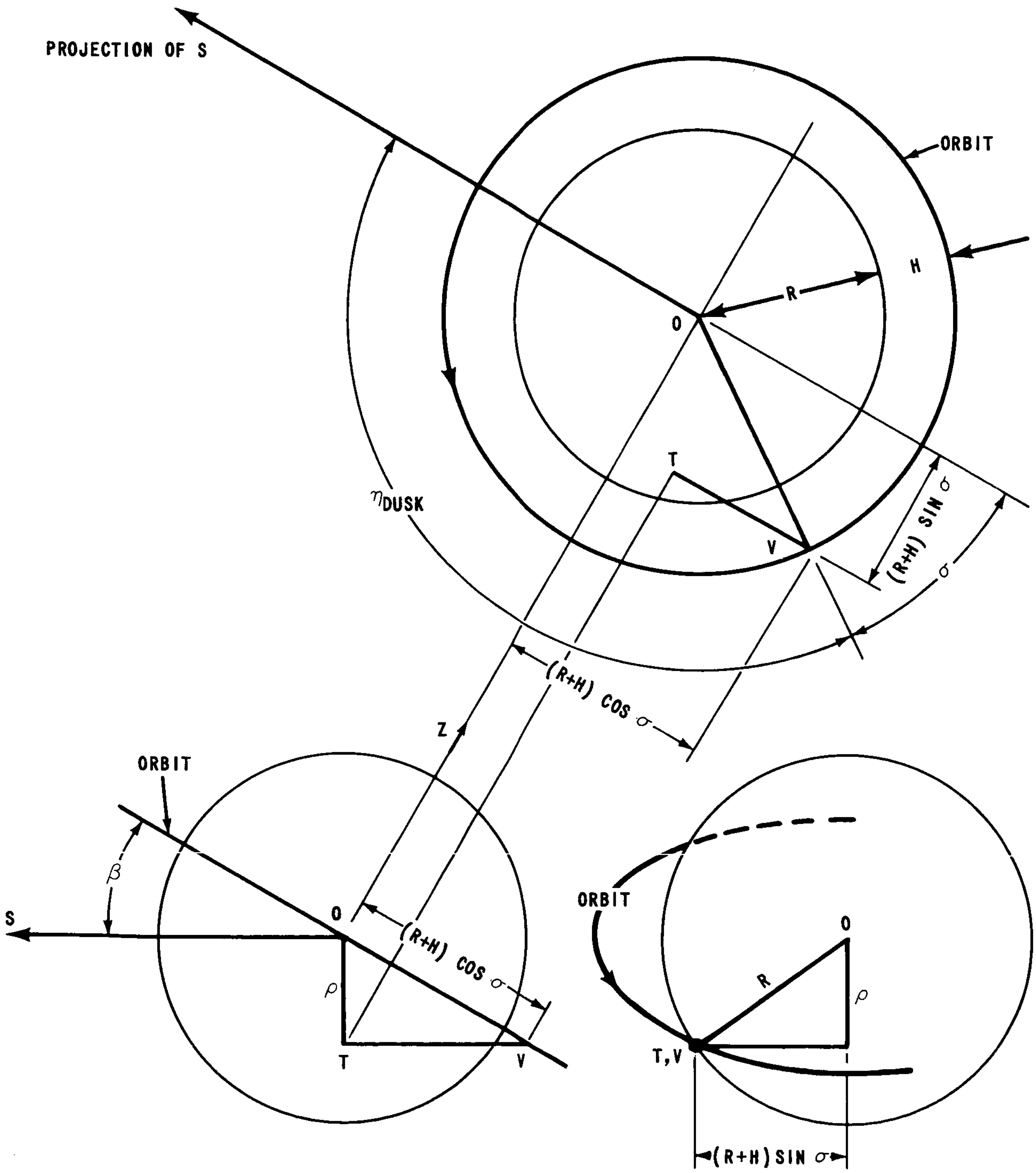

FIGURE 8: GEOMETRY FOR DETERMINING EARTH SHADOW ANGLE

T is a distance ρ below the center of the earth as measured in the S-0-Z plane perpendicular to S. V is the point on the orbit where shadow begins. R is the radius of the earth, and H is the constant altitude of the satellite. The half-shadow angle is called σ and is $\pi - \eta_{DUSK}$.

The coordinates of V in the orbital plane are shown in that view. By observing that the distance between 0 and V as seen in the S-0-Z plane is $(R + H)\cos\sigma$, we can write:

$$\rho = (R + H) \cos \sigma \sin \beta$$

When we look down the sun line, we note that:

$$\rho = \sqrt{R^2 - (R + H)^2 \sin^2\sigma}$$

By equating these two expressions for ρ, and equation for σ in terms of β, R, and H is obtained:

$$(R + H)^2 \cos^2\sigma \sin^2\beta = R^2 - (R + H)^2 \sin^2\sigma$$

$$\cos^2\sigma \sin^2\beta + \sin^2\sigma = \left(\frac{R}{R + H}\right)^2$$

$$\sin^2\sigma \cos^2\beta = \left(\frac{R}{R + H}\right)^2 - \sin^2\beta$$

$$\sin\sigma = \frac{1}{\cos\beta}\sqrt{\left(\frac{R}{R + H}\right)^2 - \sin^2\beta} \tag{23}$$

Since $\eta_{DUSK} = \pi - \sigma$,

$$\eta_{DUSK} = \pi - \sin^{-1}\left(\frac{1}{\cos\beta}\sqrt{\left(\frac{R}{R + H}\right)^2 - \sin^2\beta}\right) \tag{24}$$

For the circular orbit, $\eta_{DAWN} = - \eta_{DUSK}$.

It is possible that no shadowing occurs. This happens only when β approaches 90^o for low altitude orbits. The requirement for shadowing to occur is that $\sin\beta < \frac{R}{R + H}$. This is evident both from Figure 8 and from Equation (23).

V. ENERGY FROM SOLAR ARRAYS

The energy available from a fixed solar array in one orbit is the time integral of the solar array power output over the orbit. If we call the energy available Q and note that the angle η is directly proportional to time in a circular orbit, we can write:

$$Q = \frac{E}{2\pi}\int_{\eta \text{ beginning of sunlight}}^{\eta \text{ end of sunlight}} \cos\phi \, d\eta \tag{25}$$

where E would be the total energy if the panel faced the sun continuously and there was no shadow during an orbit. The factor 2π arises because we are integrating with η rather than time.

A. Gravity Gradient Attitude

For the gravity gradient attitude, substituting equation (19) into equation (25) yields:

$$Q_{GG} = \frac{E}{2\pi} \int_{\eta_{\text{beginning of sunlight}}}^{\eta_{\text{end of sunlight}}} (\sin\theta \cos\beta \cos(\eta + \psi) + \cos\theta \sin\beta)\, d\eta \qquad (26)$$

Evaluating the integral:

$$Q_{GG} = \frac{E}{2\pi} (\sin\theta \cos\beta \sin(\eta + \psi) + \eta \cos\theta \sin\beta) \Big/_{\eta_{\text{beginning of sunlight}}}^{\eta_{\text{end of sunlight}}} \qquad (27)$$

The lower limit, $\eta_{\text{beginning of sunlight}}$, will be either η_L defined by (22) or $\eta_{DAWN} = -\eta_{DUSK}$ defined by (24), whichever is the smaller angle. Except in the case of $\psi < -90^\circ$, the value of this lower limit will be negative. The upper limit, $\eta_{\text{end of sunlight}}$, will be either η_D defined by (21) or η_{DUSK}, whichever is the smaller angle. Except in the case of $\psi > +90^\circ$, the value of the upper limit will be positive. If both upper and lower limits on η are defined by the same kind of phenomena, i.e., if the lower limit is η_L and the upper is η_D, or if the lower is η_{DAWN} and the upper is η_{DUSK}, then the energy is not a function of ψ. In the first case:

$$Q_{GG} = \frac{E}{2\pi} (2 \sin\theta \cos\beta \sin\xi + 2\xi \cos\theta \sin\beta) \qquad (28)$$

And in the second case:

$$Q_{GG} = \frac{E}{2\pi} (2 \sin\theta \cos\beta \sin\eta_{DUSK} + 2\eta_{DUSK} \cos\theta \sin\beta) \qquad (29)$$

Therefore, in these cases there is no advantage in making ψ other than zero. The only reason for making ψ other than zero is when the solar panel system is composed of more than one planar array and the individual arrays are set at different ψ's to provide a fairly level power output on the sun side of the orbit. However, when any one panel is struck by sunlight immediately upon exiting from the earth's shadow, and then loses its view of the sun before entering the shadow (the case when upper and lower limits are of different types), the per orbit energy available from that panel will be less than if the limits are of the same type.

B. Inertial Attitude

For the inertial attitude, equation (25) becomes:

$$Q_I = \frac{E}{2\pi} \int_{\eta_{DAWN}}^{\eta_{DUSK}} \cos\phi \; d\eta \qquad (30)$$

where $\cos\phi$ is a constant. Evaluating the integral

$$Q_I = \frac{E}{2\pi} (2\eta_{DUSK} \cos\phi) \qquad (31)$$

VI. OPTIMIZATION OF Θ VS. β FOR GRAVITY GRADIENT STABILIZED ATTITUDE WITH $\psi = 0$.

The optimum θ for any β can be obtained by setting the partial derivative of Q_{GG} with respect to θ equal to zero and solving for Θ, with Q_{GG} given by either (28) or (29), whichever is applicable. If the limits on sunlight are determined by the normal to the solar array passing into the dark half-space, (28) is applicable, i.e.

$$Q_{GG} = \frac{E}{2\pi}\left[2 \sin\theta \cos\beta \sin\left\{\cos^{-1}\left(-\frac{\tan\beta}{\tan\theta}\right)\right\}\right.$$

$$\left. + 2\left\{\cos^{-1}\left(-\frac{\tan\beta}{\tan\theta}\right)\right\} \cos\theta \sin\beta\right]$$

Optimization yields

$$\frac{\partial Q_{GG}}{\partial\theta} = 0 = \frac{E}{\pi}\left[\frac{\sin\theta \cos\theta}{\cos\beta \sqrt{\sin^2\theta - \cos^2\theta \tan^2\beta}} - \frac{\sin\beta \tan\beta}{\sin\theta \sqrt{\tan^2\theta - \tan^2\beta}}\right.$$

$$\left. - \sin\beta \sin\theta \left\{\cos^{-1}\left(-\frac{\tan\beta}{\tan\theta}\right)\right\}\right] \quad (32)$$

This transcendental equation can be solved for $\theta_{optimum}$. If the limits on sunlight are determined by earth shadowing, (29) is applicable and

$$\frac{\partial Q_{GG}}{\partial\theta} = 0 = \frac{E}{\pi}\left(\cos\theta \cos\beta \sin\eta_{DUSK} - \eta_{DUSK} \sin\theta \sin\beta\right)$$

which can be solved for $\theta_{optimum}$

$$\theta_{optimum} = \tan^{-1}\left(\frac{\cos\beta \sin\eta_{DUSK}}{\eta_{DUSK}}\right) \quad (33)$$

Equations (32) and (33) have been solved for $\theta_{optimum}$ for the AAP-1/AAP-2 mission where the altitude is 270 nautical miles and the maximum β is 51.95°. The results are plotted in Figure 9.

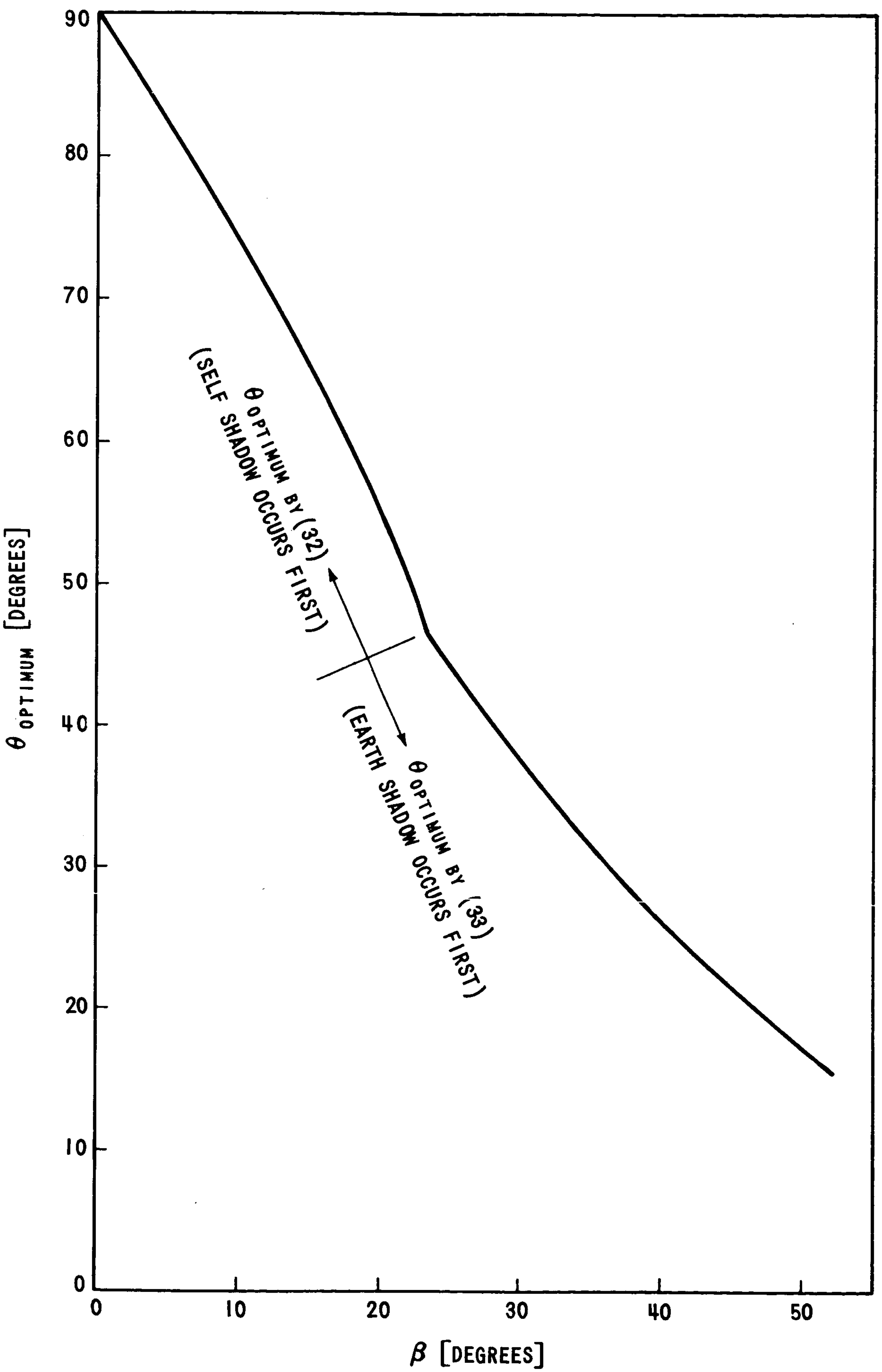

FIGURE 9: $\theta_{OPTIMUM}$ VS. β

VII. CONTINUOUS POWER FROM THE SOLAR ARRAY/BATTERY ELECTRICAL POWER SYSTEM

The ratio of continuous power available to a load from a solar array/battery EPS to the average of the solar array power output during sun exposure has been given in a recent report [4] and is representative of the solar array EPS to be used on AAP-1/AAP-2.

$$\frac{P_C}{P_{SA}} = \frac{1}{\left[1 + \frac{T_D}{T_L}\left(\frac{100}{\text{Reg B}}\right)\left(\frac{100}{B}\right)\right]\left[\frac{100 + \text{line loss}}{100}\right]\left[\frac{100}{\text{Reg V}}\right]} \tag{34}$$

where: P_C = Continuous power available

P_{SA} = Average solar array power in sunlight

$= Q/T_L$

$= Q/\left(T_o \times \frac{\eta_{\text{end of sunlight}} - \eta_{\text{beginning of sunlight}}}{2\pi}\right)$

$\frac{T_D}{T_L}$ = Ratio of dark time to light time

$$= \frac{2\pi - \eta_{\text{end of sunlight}} + \eta_{\text{beginning of sunlight}}}{+\ \eta_{\text{end of sunlight}} - \eta_{\text{beginning of sunlight}}}$$

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[4] Dunlop J. D., "AMDA/OWS Solar Array Power System Capability - Inertial Orientation", Bellcomm Memorandum for File, June 9, 1967.

Reg B = Battery regulator efficiency (93%)

B = Battery charge efficiency (65%)

Reg V = Voltage regulator efficiency (93%)

As discussed in Reference 4, using the most reasonable estimates of the efficiencies of these system components, as shown in parentheses above, and a 4% line loss, equation (35) becomes

$$P_C = \frac{P_{SA}}{(1 + 1.65 \frac{T_D}{T_L}) 1.118} \tag{35}$$

The sun oriented power output of the array, which determines the energy available from the array over one orbit, is taken as 6.07 KW. P_C has been evaluated for the gravity-gradient stabilized vehicle with panel orientation angles, θ, varying between 0 and 90° and is plotted vs. the sun line - orbital plane angle, β, in Figure 10. The function for an inertial attitude with $\phi = 0$ (i.e., $P_{SA} = 6.07$ KW) is also plotted vs. β. The upper envelope of the gravity-gradient set of curves confirms the plot of $\theta_{optimum}$ vs. β given in Figure 9.

If AAP-1/AAP-2 is flown in a gravity-gradient stabilized attitude, the value of θ which should be chosen for the solar array orientation is dependent on the versatility of the deployment and orientation mechanism, and on the flexibility of the experimentation time line. We shall first consider the case where the experiment time line is flexible. By this we mean that the Workshop experiment requiring the greatest power would be run on the days when β is such that the continuous power from the solar panel-secondary battery system is highest. Our objective is then to set θ to obtain the maximum power over the mission.

The highest average continuous power available over a mission would be obtained if the angle θ could be varied during the mission, possibly by daily increments, to follow the curve of Figure 9. The continuous power would then be the value given by

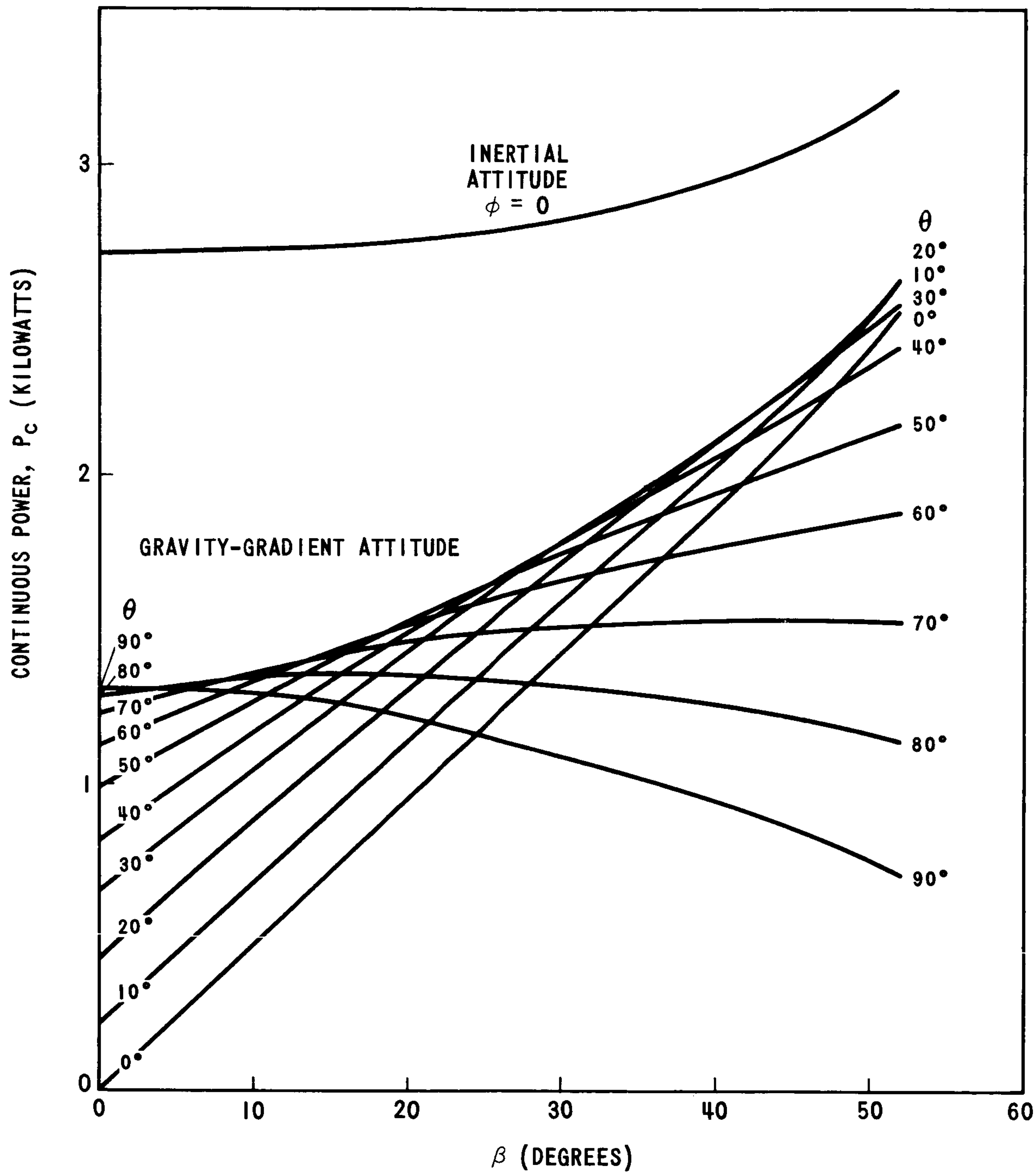

FIGURE 10: SOLAR ARRAY/BATTERY BUS POWER vs β ANGLE

the upper envelope of Figure 10 at any particular value of β. Neglecting spacecraft shadowing, this method would yield 48% of the power available with the inertial attitude at $\beta = 0$, and 81% of the power available with the inertial attitude at $\beta = 51.95^\circ$. At the mean value of $|\beta|_{avg} = 20.44^\circ$ the power is 57% of that with the inertial attitude. If the panel orientation mechanism is such that θ cannot be changed after launch, then a θ chosen from Figure 9 for the average of the β angle over the 28-day mission will yield almost the maximum power. A panel orientation angle chosen in this manner will not provide the absolute maximum average power because the power for a given θ is not a linear function of β. However, the difference in continuous power output for a θ chosen in this fashion, and the absolute maximum obtainable from a fixed array will be only a few watts.

If launch time is set by day and time-of-day, the $|\beta|_{avg}$ can be calculated, as was done in generating Figure 4. For instance if launch is to occur at noon on October 24, $|\beta|_{avg}$ will be about $17\frac{1}{2}^\circ$ for the mission, and by Figure 9 we find that θ optimum will be about 61°. Notice that a slip in the launch time of one to several days will not appearably affect this result, as long as the time of day of launch remains 1200 hours. If launch time is set by time-of-year only, and there are to be no constraints on the time-of-day, then θ should be picked to optimize power for the daily mean of $|\beta|_{avg}$ for that time of year. In the case of mid-October we would pick $\theta = 55^\circ$ from Figure 9 corresponding to $\beta = 20^\circ$. If there is no opportunity to optimize θ for launch time, it should be set for the mean of $|\beta|_{avg}$ over the year. For $|\beta|_{avg} = 20.44^\circ$, $\theta = 54^\circ$.

If the power profile is inflexible, and a fairly level power output is desired for any β, then θ should be set at about 75°. This is evident for Figure 10. With this orientation angle, the continuous power will vary between 1.27 KW and 1.40 KW.

The average of the continuous power output over a 28-day missions launched at about 1000 different times have been calculated for the following five cases:

1. Inertial Attitude, $\phi = 0$
2. Gravity-Gradient Attitude, $\theta = 75^\circ$, $\psi = 0$
3. Gravity-Gradient Attitude, $\theta = 60^\circ$, $\psi = 0$
4. Gravity-Gradient Attitude, $\theta = 54^\circ$, $\psi = 0$
5. Gravity-Gradient Attitude, $\theta = 45^\circ$, $\psi = 0$

The maximum, minimum, mean, and variance of these average values of continuous powers are given in the following table.

	AVERAGE CONTINUOUS POWER OVER 28 DAYS (KILOWATTS)				
	INERTIAL ATTITUDE $\phi=0$	GRAVITY-GRADIENT ATTITUDE			
		$\theta=75^\circ$	$\theta=60^\circ$	$\theta=54^\circ$	$\theta=45^\circ$
MAXIMUM	2.94	1.38	1.69	1.77	1.89
MINIMUM	2.73	1.33	1.31	1.29	1.21
MEAN	2.80	1.36	1.47	1.49	1.49
VARIANCE	.062	.015	.103	.134	.186

The 75° solar panel orientation angle gives the lowest variance, indicating that this provides a fairly level power output regardless of β. The case of $\theta = 54^\circ$ gives almost the maximum mean of average power for a random launch time over the year. The difference between the power available at this orientation angle and the maximum possible power with a fixed θ is less than 10 watts.

In this study, the average power output during an individual mission and at a fixed θ has been calculated by averaging the power obtained at the various β's that occur at fixed intervals of mission time. It is worthwhile to examine the difference between the average power determined in this manner, and the power available at the average of $|\beta|$ for the same mission. A specific example is required. The $|\beta|_{avg}$ for a mission launched October 30 at 0300 will be 18.38°, and with θ set at 54°, the average continuous power will be 1.449 KW. An individual orbit with β = 18.38° and θ = 54° will give a power of 1.466 KW. The difference is 17 watts or about 1.2%. The simpler method of calculating the power at the average value of $|\beta|$ gives a good, but generally unconservative, approximation to the actual average power over a mission. The fact that such an estimate is in general unconservative can be verified by noting the decreasing slope of the curves in Figure 10 with increasing β with θ greater than 20°.

VIII. SUMMARY

In the initial sections of this report the expression for the sine of the angle between the sun line and the orbital plane, β, has been developed in terms of a set of constants fixed by launch day and time-of-day times harmonic functions of mission time. Expressions for continuous power output of a solar array /battery electrical power system as functions of array orientation with respect to an orbiting satellite, satellite attitude, and β have also been obtained. Statistical analyses of the absolute value of β and its trigonometric functions, and of continuous power output have been performed numerically to obtain average values of these quantities over the duration of the AAP-1/AAP-2 mission. Maximums, minimums, means and variances of these averages have been found to study the power system capabilities for a random launch time. This study of the averages over the mission is a more valid base for comparing power system configurations and flight attitudes than a study of outputs at the limits of $|\beta|$, 0 and 51.95°, which occur infrequently, and perhaps never, over the course of a mission.

The statistical study of $|\beta|_{avg}$ over a 28-day mission has yielded the values of 35.28°, 9.54°, and 20.44° for the maximum, minimum, and mean respectively. The average continuous power output for a solar array battery system on a satellite held inertially with the array sun orientated is a function only of $|\beta|$. For the electrical power system considered (a 6.07 KW array), the mission average continuous output of the system will vary between 2.73 KW and 2.94 KW and its mean for a random launch time will be 2.80 KW. Although these absolute values could vary by virtue of the EPS design, ratios between them are independent of the system.

If the satellite is passively stabilized with gravity-gradient induced torques, then additional variables describing the array orientation with respect to the vehicle or orbit axes enter the calculations. The angle between the plane of the array and the orbital plane, θ, can be set to optimize the power output of the solar array. The second angle required to establish array orientation is ψ, which is the angle between the local vertical and the projection of the normal to the array in the orbital plane. A value of ψ other than zero might be used if the solar array system consists of several planar arrays and they are positioned in different orientations to provide a fairly level power output over the sunlit side of the orbit. Another possible constraint might require a value of ψ other than zero. If the Workshop solar array orientation in the AAP-3/AAP-4 mission must be optimized for that mission's inertial attitude, <u>and</u> there is no provision to change panel orientation between the two sets of missions, then the panels would be parallel to the Workshop center line. At $\beta = 0$, there will be no output if θ is zero. With this configuration, a θ other than zero automatically makes $\psi = \pm 90^{\circ}$. This constraint would severely limit the capability of the system at low β angles; the array output would be only slightly greater than half that available with a large θ and $\psi = 0$, and the continuous power available to the load would be much less than half.

Configurations for gravity-gradient stabilized spacecraft which allow ψ to be kept at zero and θ to be set for the mission will yield average powers that increase with the versatility of the orientation mechanism. If θ can be varied during the mission as a function of β to maximize the continuous power output, then from 48% to 81% of the continuous power output of the inertial attitude will be available, with 57% available at the yearly mean of the average β angle. If θ cannot be varied after launch, it can be set for the average β for the mission with increasing average power outputs obtained with increasing constraints on launch time. With no constraints on launch time and with θ set to optimize power output at the yearly mean of β, the yearly mean of the mission average continuous power output will be 53% of the yearly mean of the output with an inertial orientation. If a fairly level power output is required, a θ of 75° provides a continuous power that is not a strong function of β. However, the yearly mean of the output at this orientation angle drops to 48% of the yearly mean with an inertial attitude.

Although this analysis of continuous power output has been done for cases where all solar cells are in a single plane, results for a set of smaller panels at different orientations with respect to the spacecraft can be obtained by summing results from analyses of each of the smaller panels.

IX. ACKNOWLEDGEMENT

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